Weighted Model Integration

Weighted Model Integration (WMI) is an exact probabilistic inference method for probabilistic models. Inference in probabilistic programs can be reduced to WMI.

**Rectangular Decomposition Methods** use an SMT solver to sample axis-aligned rectangles from the region of integration. The probabilities of these rectangles are easy to compute.

Rectangular Decomposition can suffer from inefficient sampling.

**Orthogonal Transformations to the Rescue!**

**The Big Idea:**
Can we improve a formula’s sampling efficiency by rotating or reflecting its variables?

**Q:** What distributions will this work for?
**A:** Skitovitch-Darmois Theorem $\implies$ Only Gaussians.

**Q:** Rotations may introduce irrational numbers, which are bad for SMT solvers. How do we avoid that?
**A:** Use Pythagorean triples to generate rational Givens rotations:

We compose multiple Givens rotations to obtain a rotation matrix that (approximately) aligns the formula’s faces with the axes.

Our method is related to QR-factorization by Givens rotations.

**Q:** Does this actually improve efficiency?
**A:** Yes. Rotations improved sampling efficiency for 10 of 12 probabilistic program benchmarks. In some programs, rotations made little difference either way; in others it led to dramatic improvements. For example:

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